Path Planning and Steering Control for an Automatic Perpendicular Parking Assist System

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***Abstract*— This paper considers the perpendicular reverse parking problem of front wheel steering vehicles. Relationships between the widths of the parking aisle and the parking place, as well as the parameters and initial position of the vehicle for planning a collision-free reverse perpendicular parking in one maneuver are first presented. Two types of steering controllers (bang-bang and saturated *tanh*-type controllers) for straight-line tracking are proposed and evaluated. It is demonstrated that the saturated controller, which is continuous, achieves also quick steering avoiding chattering and can be successfully used in solving parking problems. Simulation results and first experimental tests confirm the effectiveness of the proposed control scheme.**

I. Introduction

The perpendicular parking is the most efficient and economical since it accommodates the most vehicles per linear meter [1], and is especially effective in long term parking areas. Due to the special constraint environments, much attention and driving experience is needed to control the vehicle, and this parking maneuver may be a difficult task. For this reason, automated operation attracts significant attention from research view point, as well, and from the automobile industry. One of the difficulties in achieving automatic parking is the narrow operating place for collision-free motion of the vehicle during the parking maneuver, and planning of optimal trajectories is often used in the applications. In [2], an optimal stopping algorithm was designed for parking using an approach combining an occupancy grid with planning optimal trajectories for collision avoidance. The geometry of the perfect parallel parking maneuver is presented in [3]. In [4], a practical reverse parking maneuver planner is given. A trajectory planning method based on forward path generation and backward tracking algorithm, especially suitable for backward parking situations is reported in [5]. A car parking control using trajectory tracking controller is presented in [6]. In [7], a saturated feedback control for an automated parallel parking assist system is reported. In recent years, automatic parking systems have been also developed by several automobile manufacturers [9, 10].

In this paper, we focus on geometric collision-free path planning, and feedback steering control for perpendicular reverse parking in one maneuver. Geometric path planning based on admissible circular arcs within the available parking spot is presented in order to steer the vehicle in the direction of the parking place in one maneuver. Two steering controllers (bang-bang and saturated *tanh*-type) for path tracking are proposed and evaluated. The rest of the paper is organized as follows: In Section II, geometric considerations for planning perpendicular reverse parking in one maneuver are presented. In Section III, two feedback steering controllers are proposed. Simulation results and first experimental tests are reported in Section IV. Section V concludes the paper.

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II. Geometric Considerations for Collision-Free Perpendicular Parking in One Maneuver

A. Vehicle Model

In this paper, a rectangular model of a front-wheel passenger vehicle is assumed. The vehicle parameters which affect the parking maneuver, as well as the parameter values used in the simulations, are presented in Table I.

TABLE I. **Vehicle Parameters**

Vehicle parameters	*Notation*	*Value*
Longitudinal vehicle base	l	*2.6m*
Wheel base	b	*1.8m*
Distance between the front axle and the front bumper	l_1	*0.94m*
Distance between the rear axle and the rear bumper	l_2	*0.74m*
Maximum steering angle	α_{max}	*π/6rad*

B. Collision-Free Path Planning with a Constant Turning Radius

The geometry of the reverse perpendicular parking in one collision–free maneuver is shown in Fig. 1. In the perpendicular parking scenario considered in this paper, the vehicle starts to move backward from an initial position *1* in the parking aisle, with constant steering angle α_c, which may be smaller than the maximum steering angle ($|\alpha_c| \le |\alpha_{max}|$), and has to enter in the parking place (position *2*) without colliding with the boundary c_1 of parking lot L1 and boundaries c_2, and c_3 of parking lot L2. In position 2 the orientation of the vehicle is parallel with respect to the parking place. After that, the vehicle continues to move backward in a straight line into the parking place until it reaches the final position *3* (Fig. 1). Assuming a circular motion of the vehicle (with turning radius ρ_c), with center *O* (Fig. 1). The radius ρ_c is calculated from the formula

$$\rho_c = \frac{l}{\tan \alpha_c}. \quad (1)$$

The boundaries of the turning path during the perpendicular parking are determined by the dimensions of the traces (circular arcs) formed by the left corner of the front bumper B_2 with radius r_{B2}, the left corner of the rear bumper B_4 with radius r_{B4}, and the end of the rear wheel axle C_1, respectively, as shown in Fig.1. Since the vehicle executes a plane rotation, the trajectories of these points form arcs of concentric circles.

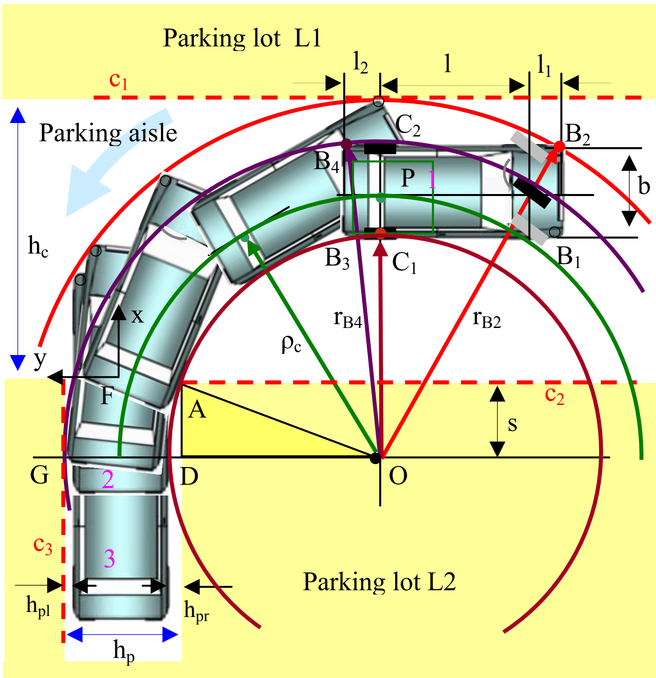

Figure 1. Geometry of the collision-free perpendicular parking maneuver

From the ΔOC_2B_2, applying the Pythagorean Theorem, we obtain an expression for the radius r_{B2} of the circular arc traced by the left corner of the front bumper B_2 in terms of the vehicle parameters l, l_1, b , and the turning radius ρ_c, as follows

$$r_{B2} = OB_2 = \sqrt{(l + l_1)^2 + \left(\rho_c + \frac{b}{2}\right)^2}\,. \tag{2}$$

From the ΔOC_2B_4, we determine the radius r_{B4}, of the circular arc traced by the left corner of the rear bumper B_4

$$r_{B4} = OB_4 = \sqrt{l_2^2 + \left(\rho_c + \frac{b}{2}\right)^2}\,. \tag{3}$$

We assign an inertial frame Fxy attached to the parking place, where the center F is placed in the middle between the borders of the parking place, which has its y-axis aligned with the boundary c_2 of parking lot L2, as shown in Fig. 1. Let O denotes the center of rotation of the vehicle (the Instantaneous Center of Rotation (ICR)) when it starts the parking maneuver with constant steering angle α_c. Depending on the sign of x-coordinate of ICR (point O) with respect to the Fxy frame, i.e., the offset s (Fig. 1), different formulas can be derived in order to determine the required width h_p of the parking place and the width of the parking aisle (the corridor) h_c as functions of s in order to ensure collision-free perpendicular parking in one maneuver. We consider right turning of the car in the following two cases:

- The ICR O belongs to the interval: $s \in [-(\rho_c - b/2),\quad 0]$

The lower value of the interval corresponds to the case when the right side of the vehicle B_1B_3 (Fig.1) lies on the boundary line c_2 of parking lot L2.

In order to avoid collision between the left corner B_2 of the front bumper with the boundary c_1 of L1 (Fig. 1), using (2), we obtain an expression for the width of the parking aisle h_c, as follows

$$h_c = r_{B2} - |s| = \sqrt{(l + l_1)^2 + \left(\rho_c + \frac{b}{2}\right)^2} - |s|\,. \tag{4}$$

The function h_c = f(s) defined by (4) is linear in s, positive and monotonically increasing in the above-mentioned closed interval for s. Therefore, it takes its minimum and maximum values at the ends of this interval.

To avoid a collision between the right point C_1 of the rear axle with the vertex A of obstacle L2, from the ΔOAD, applying the Pythagorean Theorem, the distance OD (Fig. 1) is calculated as follows

$$OD = \sqrt{\left(\rho_c - \frac{b}{2}\right)^2 - s^2}\,. \tag{5}$$

In order to avoid a collision between the left corner B_4 of the rear bumper with the edge c_3 of the parking place, using (3) and (5), the following expression for the width h_p of the parking space is obtained

$$h_p = r_{B4} - OD = \sqrt{l_2^2 + \left(\rho_c + \frac{b}{2}\right)^2} - \sqrt{\left(\rho_c - \frac{b}{2}\right)^2 - s^2}\,. \tag{6}$$

The function h_p = f(s) defined by (6) is continuous on the closed interval of s mentioned above. This function is differentiable on the open interval $s \in (-(\rho_c - b/2),\quad 0)$, and its derivative is given by

$$\frac{\partial h_p}{\partial s} = \frac{s}{\sqrt{\left(\rho_c - \frac{b}{2}\right)^2 - s^2}} < 0\,. \tag{7}$$

Therefore, the function h_p = f(s) is strictly decreasing on the closed interval $[-(\rho_c - b/2),\ 0]$. The maximum and minimum values of h_p can be found by replacing in (6) the boundary values of the interval: $s = -(\rho_c - b/2)$ and $s = 0$.

- The ICR O belongs to the interval: $s \in [0, l_2]$

The upper bound l_2 corresponds to the case when the rear bumper lies on the Fy-axis at the instant when the orientation of the vehicle is parallel to the parking place.

In order to avoid a collision between the left corner B_2 of the front bumper with the boundary c_1 of L1, using (2), we obtain an expression for the width of the parking aisle h_c

$$h_c = r_{B2} + s = \sqrt{(l + l_1)^2 + \left(\rho_c + \frac{b}{2}\right)^2} + s \cdot \quad (8)$$

Again, the function h_c = f(s) defined by (8) is linear in s, positive and monotonically increasing in the above-mentioned close interval of s. Therefore, it takes its minimum and maximum values at the ends of this interval.

To avoid a collision between the left corner B_4 of the rear bumper with the edge c_3 of the parking place, and between the right point C_1 of the rear vehicle axle with the vertex A of obstacle L2, we obtain the following expression for h_p

$$h_p = \sqrt{l_2^2 + \left(\rho_c + \frac{b}{2}\right)^2 - s^2} - \left(\rho_c - \frac{b}{2}\right) \cdot \quad (9)$$

The function h_p = f(s) defined by (9), is continuous on the closed interval of $s \in [0, l_2]$. This function is differentiable on the open interval $s \in (0, l_2)$ and the derivative is

$$\frac{\partial h_p}{\partial s} = -\frac{s}{\sqrt{l_2^2 + \left(\rho_c + \frac{b}{2}\right)^2 - s^2}} < 0 \cdot \quad (10)$$

Therefore the function is strictly decreasing on the closed interval $s \in [0, l_2]$. The maximum and minimum values of h_p can be found by replacing the limit values $s = 0$ and $s = l_2$ of the interval, respectively, in the expression (10). It should be noted that for $s = 0$, the two functions defined by (6) and (9) take the same maximum value. For $s = l_2$, the function h_p = f(s) takes minimum value, which is exactly the width b of the vehicle.

From a practical point of view, it is important to determine the starting positions of the vehicle for parking without collision in one maneuver in the case when the widths h_c and h_p of the parking aisle and the parking space, respectively, are specified in advanced. Suppose that the widths of the parking aisle and the parking place are set as $h_c = h_{cd}$ and $h_p = h_{pd}$, respectively, and also that $h_{cd} < r_{B2}$. In this case, from (2) and (4), it follows that

$$-|s|_{\max} = h_{cd} - r_{B2} \cdot \quad (11)$$

From (3) and (6), we obtain a formula for the minimum value of s as follows

$$-|s|_{\min} = -\sqrt{\left(\rho_c - \frac{b}{2}\right)^2 - (r_{B4} - h_{pd})^2} \cdot \quad (12)$$

Simulation results were performed to illustrate the relationships between the widths h_c and h_p of the parking aisle and the parking space, respectively, as functions of the offset s in the interval [-(ρ – $b/2$), 0] by using parameters of the test vehicle (Table I) with $\alpha_c = \alpha_{max}$, ($\rho_c = \rho_{min}$). The values of h_c and h_p, (h_{cd} and h_{pd}), were chosen as follows: $h_{cd} = 6m$ and $h_{pd} = 2.4m$.

As seen from Fig.2, the function h_p = f(s) (the solid blue line) decreases in the interval and converges to $b=1.8m$ (the red dotted line), which is exactly the length of the wheel base of the vehicle. Meanwhile, the graph intersects the horizontal line for the assigned value of $h_{pd} = 2.4m$ (the blue dotted line) at $s = -|s|_{min} = -1.91m$, which is the minimum value of s obtained from (12) for collision-free parking. In order to park the vehicle in one maneuver for $s = -|s|_{min} = -1.91m$, from (8), the required minimum width h_c of the parking aisle is obtained to be $h_c = 4.55m$ which is less than the specified value of $h_{cd} = 6m$.

The function h_c = f(s) (the green solid line) increases linearly in the interval and the graph intersects the horizontal line for the assigned value of $h_{cd} = 6m$ (the green dotted line) at $s = -|s|_{max} = -0.46m$, which is the maximum value of s, obtained from (11). For $s = -|s|_{max} = -0.46m$, from (6), the required minimum width h_p of the parking place has to be h_p = 1.88m, which is less than the assigned value of $h_{pd} = 2.4m$.

Therefore, given specified values $h_c = h_{cd} = 6m$ and $h_p = h_{pd} = 2.4m$ for the parking aisle and the parking space, respectively, for collision-free parking, the offset s can take values in the interval [- $|s|_{min}$, - $|s|_{max}$] = [$-1.91m$, $-0.46m$], where the boundary values are determined by (12) and (11), respectively.

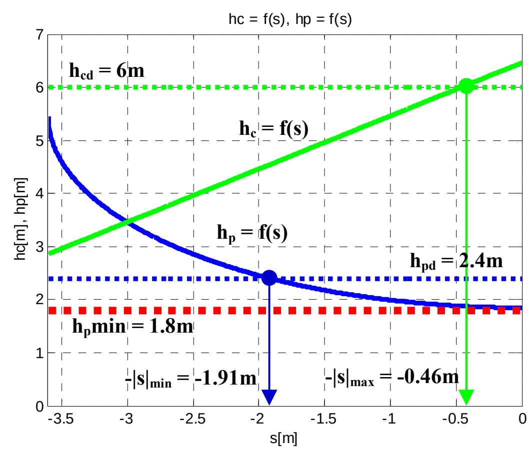

Figure 2. Collision-free interval for s

The distances between the car and the boundaries of the parking space h_{pl} and h_{pr} (Fig. 1), when the vehicle is parallel to the parking space, are determined as follows

$$h_{pr} = \left(\rho_c - \frac{b}{2}\right) - \sqrt{\left(\rho_c - \frac{b}{2}\right)^2 - s^2} \,, \quad (13)$$

$$h_{pl} = h_{pd} - b - h_{pr} \,. \quad (14)$$

From the simulations, for $s = -|s|_{min} = -1.91m$, the obtained values of h_{pr} and h_{pl} are $h_{pr} = 0.55m$ and $h_{pl} = 0.05m$.

From a practical view point, it is better to park the car symmetrically with respect to the boundaries of the parking place, since it is not very wide. For this end, we calculate the minimum value of the offset $s = s_m$, in order to park the vehicle symmetrically in the center of the parking space (Fig. 3). We set

$$h_{ps} := h_{pr} = h_{pl} = \frac{h_{pd} - b}{2} . \quad (15)$$

From the ΔOAD (Fig. 3), the distance OD is determined as

$$OD = \sqrt{\left(\rho_c - \frac{b}{2}\right)^2 - s_m^2} . \quad (16)$$

Since the turning radius can be expressed as

$$\rho = \frac{b}{2} + h_{pr} + OD , \quad (17)$$

and substituting h_{pr} from (13) and OD from (16) into (17), we arrive to an expression for s_m, as follows

$$-|s|_m = -\sqrt{\left(\rho_c - \frac{b}{2}\right)^2 - \left(\rho_c - \frac{h_{pd}}{2}\right)^2} . \quad (18)$$

The new offset $-|s_m|$ is bigger than those given by (12) ($-|s|_m > -|s|_{min}$). In the simulation results, $-|s|_m = -1.44m > -1.91m$. In general, it must be checked whether the new offset $-|s|_m$ is smaller than $-|s|_{max}$ given by (11). If it is the case, the car can park symmetrically without collision in reverse when s is at least $s = -|s|_m$. In this case, however, the boundary c_3 of the parking place will not be tangent to the arc of circle traced by point B_4 of the left corner of the rear bumper; nevertheless, point A (vertex A of obstacle L2) will lie again on the arc of circle traced by point C_1 of the rear vehicle axle. Therefore, given specified dimensions of the parking aisle and parking place $h_c = h_{cd}$ and $h_p = h_{pd}$, respectively, the offset s can take values in the closed interval $-|s| \in [-|s|_m, -|s|_{max}]$, where $-|s|_m$ and $-|s|_{max}$ are determined from formulas (18) and (11), respectively, (Fig.3).

Hence, in order to perform reverse perpendicular parking in one maneuver and to place the vehicle symmetrically in the parking place, the starting position, i.e., the reference point P of the vehicle has to be on any one of the arcs of circles with radius ρ of center $O(x_O, y_O)$, where $x_O \in [-|s|_m, -|s|_{max}]$ and $y_O = -\rho_c$, with respect to an inertial frame Fxy attached to the parking place. The initial orientation has to be tangent to the arc (Fig. 3). The reference path of the parking maneuver consists of two parts. The first one is a circular arc with center O connecting the staring position of the vehicle and the tangent point T between the arc and the x-axis of Fxy. At that point, the car will be parallel to the parking place. The second part of the reference path is a straight line along the y-axis of the coordinate frame Fxy between point T and the goal position G of the parking place, where point G lies on the x-axis of Fxy, (Fig. 3).

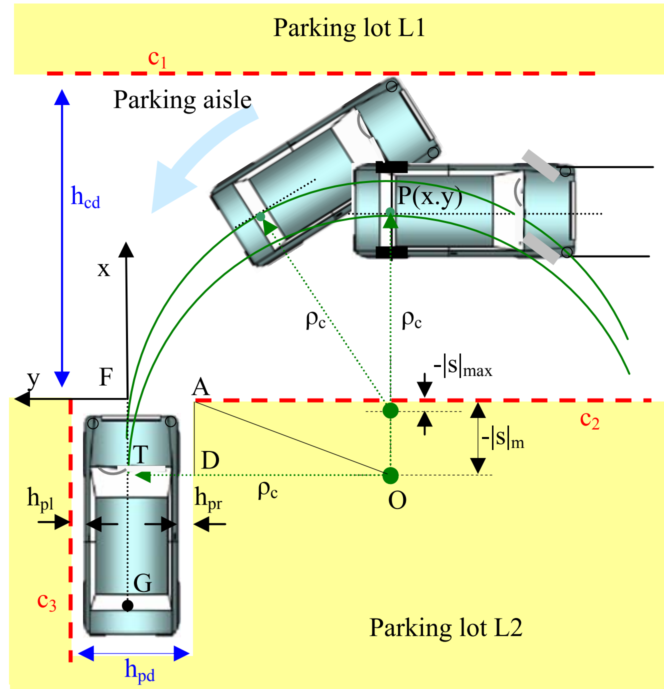

Figure 3. Geometry of collision-free perpendicular parking

III. Steering Control

For a low speed motion, which is the case of the parking maneuver, we assume that the wheels of the vehicle roll without sliding, and the velocity vectors are in the direction of the orientation of the wheels. We consider a simplified (bicycle model) of the vehicle, where the front and rear wheels are replaced by two virtual wheels, placed at the longitudinal axis of the vehicle. An inertial coordinate system is attached to the parking place (Fig. 3). The coordinates of the reference point P in Fxy are denoted by (x_P, y_P). The orientation of the vehicle θ is defined as an angle between the x-axis of Fxy and the longitudinal vehicle base. The front wheel steering angle is denoted by α. The equations of motion of the vehicle in the plane have the form [7]

$$\begin{aligned} \dot{x}_P &= v_P \cos\theta \\ \dot{y}_P &= v_P \sin\theta , \\ \dot{\theta} &= \frac{v_P}{l} \tan\alpha \end{aligned} \quad (19)$$

where v_P is the velocity of point P. We consider a practical stabilization of the vehicle in the parking place. Our approach is based on controlling the motion of the vehicle along a straight line (the x-axis of Fxy) passing through the goal point G (Fig.3) in the parking place and aligned with the orientation of the place with velocity of the car, which is dependent of the distance between the vehicle and the goal position [7]. Since the reference path for the first part of the parking maneuver is a circular arc, first a bang-bang controller is proposed, where the front wheel steering angle is constrained by magnitude and takes only two constant values. As a consequence, the vehicle trajectories represent circular arcs. However, in practice, due to the discontinuity of the control law, an undesirable behavior of the system (chattering) will occur when the position of the vehicle is in the vicinity of the tracking line, and the orientation error is also small. In order to avoid the chattering, a saturated control based on hyperbolic tangent function is also proposed, which is constrained by magnitude, but the control function is continuous.

A. Bang-Bang Control

In this paper, we propose a bang-bang control in the case when the vehicle is moving backward, ($v_P = -|v_P| < 0$). The vehicle has to track a straight line which coincides with the x-axis of coordinate frame *Fxy*. The design of the control low is based on the second and third equations of (19). The steering angle of the front wheels is constraint and takes values $\pm\alpha_c$. For brevity of exposition, we will present the final form of the bang-bang control. The control design procedure for backward driving of the vehicle is similar to those presented in [8], but the form is slightly different, since the vehicle velocity has negative sign. The bang-bang controller for backward driving has the form

$$u = \begin{cases} u & \text{if} \quad y_P > 2\dfrac{l}{\tan\alpha_c}\sin\dfrac{\theta}{2}\left|\sin\dfrac{\theta}{2}\right| \\ & \text{or} \quad y_P = 2\dfrac{l}{\tan\alpha_c}\sin\dfrac{\theta}{2}\left|\sin\dfrac{\theta}{2}\right| \quad \text{and} \quad y_P > 0 \\ -u & \text{if} \quad y_P < 2\dfrac{l}{\tan\alpha_c}\sin\dfrac{\theta}{2}\left|\sin\dfrac{\theta}{2}\right| \\ & \text{or} \quad y_P = 2\dfrac{l}{\tan\alpha_c}\sin\dfrac{\theta}{2}\left|\sin\dfrac{\theta}{2}\right| \quad \text{and} \quad y_P < 0 \end{cases}, \qquad (20)$$

where

$$u = \frac{\tan\alpha_c}{l}\,. \qquad (21)$$

B. Saturated Control

In order to avoid chattering in the system when a pure bang-bang control is used, we propose a differentiable saturation in the form of hyperbolic tangent (*tanh*(.)) constraint. This function is bounded by ± 1. Also $tanh(x) \geq 0$ if $x \geq 0$, and $tanh(x) < 0$ if $x < 0$. *Tanh*(x) is close to the signum function, when in $tanh(K_t x)$ the gain K_t is large, as shown in Fig. 4.

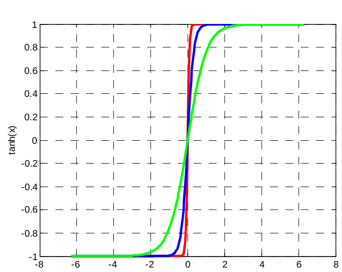

Figure 4. The function $tanh(K_t x)$ for $K_t = 1,3$, and 10

We propose the following feedback bounded steering controller

$$\alpha = a\tan\left[lu\tanh(K_t v)\right], \qquad (22)$$

where u is given by (21),

$$v = K(\theta - a_0 y_P)\,, \qquad (23)$$

and K_t, K and a_0 are positive constants.

IV. Simulation and Experimental Results

Simulation results using MATLAB are presented to illustrate the effectiveness of the proposed steering controllers for perpendicular reverse parking in one maneuver. The parameters of the vehicle are given in Table I. For the simulations, the constant steering angle of the front wheels was chosen to be $\alpha_c = \alpha_{max} = \pi/6rad$. Using (1), for the minimum turning radius ρ is obtained the value of $\rho=4.5m$. The parking aisle h_{cd} was $6m$ wide, while the width of the parking place h_{pd} was $2.4m$. The initial coordinates of the vehicle reference point P with respect to the inertial frame *Fxy* attached to the parking place were $(x_P(0), y_P(0)) = (3.5m, -4.5m)$. In this case, the offset s is equal to $s = -1m$ and belongs to the interval $[-|s|_m, -|s|_{max}] = [-1.44m, -0.46m]$ for symmetric parking in one maneuver. The initial orientation of the vehicle was chosen to be $\theta(0) = -\pi/2rad$. The initial coordinates of the vehicle reference point P with respect to an inertial frame *Gxy* with center placed in the goal position G of the vehicle in the parking place (Fig. 3), and which has its x-axis aligned with the x-axis of *Fxy* are $(x_P(0), y_P(0)) = (7.5m, -4.5m)$. The maximum value of the vehicle velocity was chosen to be $|v_P| = 0.3m/s$. The values of the saturated *tanh*-type controller were $K_t = 8, K = 5.85, a_0 = 0.17$.

Starting from identical initial conditions, the planar paths of the vehicle using bang-bang control and saturated (*tanh*-type) control are presented in Fig. 5. As seen from the simulation, the vehicle trajectories are quite similar. This result shows that the saturated control can be used instead of bang-bang control in order to steer the vehicle into the parking place according to the geometrical considerations for collision-free reverse perpendicular parking in one maneuver presented in Section II.

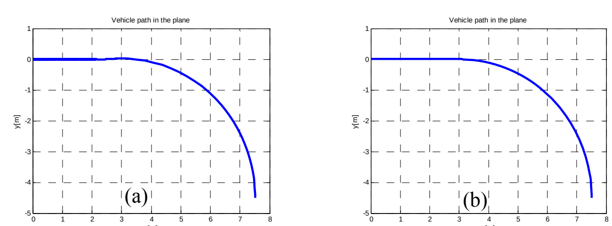

Figure 5. Perpendicular parking: Planar paths of the vehicle using bang-bang control (a) and saturated control (b).

Evolution in time of the front-wheel steering angle by using bang-bang control and saturated control is presented in Fig. 6. The simulation results show the advantage of the saturated control: the chattering occurring using bang-bang-control, when the position of the vehicle is in the vicinity of the tracking line, and the orientation error is also small, is avoided.

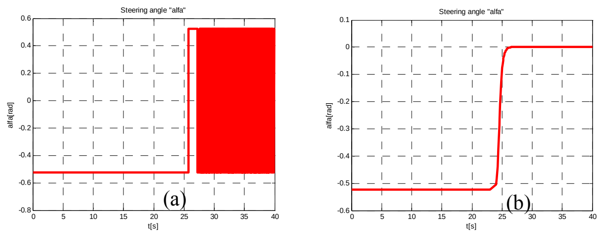

Figure 6. Perpendicular parking: Evolution in time of the front-wheel steering angle using bang-bang control (a) and saturated control (b)

An animation of the perpendicular reverse parking in one maneuver using saturated *tanh*-type steering control is shown in Fig. 7.

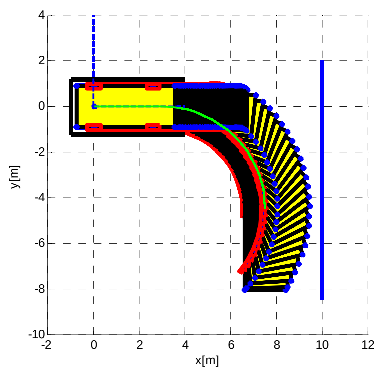

Figure 7. Perpendicular reverse parking using saturated control

The saturated *tanh*-type controller has been implemented on an experimental automatic electric vehicle CyCab and initial tests of perpendicular reverse parking has been initialized (Fig. 8). In the first tests, only information from the encoders mounted on the wheels were used for determining the position of the vehicle with respect to an inertial frame attached to the goal position into the parking place. The dimensions of the CyCab are: $l = 1.2m$; $b = 1.2m$; $l_1 = l_2 = 0.35m$; $\alpha_c = \alpha_{max} = \pi/6rad$ and $\rho_c = \rho_{min} = 2.08m$. The assigned values for the parking aisle and the parking place were chosen to be $h_{cd} = 3m$ and $h_{pd} = 2m$, respectively. For symmetric parking into the parking place, according to (18) and (11) the offset s can take values in the closed interval $-|s| \in [-|s|_m, -|s|_{max}] = [-1.01m, -0.095m]$. For the experiment shown in Fig. 8, the initial coordinates of the vehicle with respect to Gxy with center placed at the goal position in the parking place were approximately $(x_P(0), y_P(0)) = (3m, -2.1m)$. The first experiments confirm the effectiveness of the proposed controller.

V. Conclusion

In this paper, the problem of perpendicular reverse parking of front wheel steering vehicles was considered. Geometric considerations for collision-free perpendicular parking in one reverse maneuver were first presented, where the shape of the vehicle and the parking environment were expressed as polygons. Relationships between the widths of the parking aisle and parking place, as well as the parameters and the initial position of the vehicle have been given, in order to plan a collision-free maneuver, in the case, when the car has to be symmetrically positioned into the parking place. Two types of steering controllers (bang-bang and saturated controllers) for straight-line tracking have been proposed and evaluated. It was demonstrated that, the saturated *tanh*-type controller, which is continuous, was able to achieve also quick steering avoiding chattering and can be successfully used in solving parking problems. Simulation results and the first experiments with a test vehicle confirm the effectiveness of the proposed control scheme.

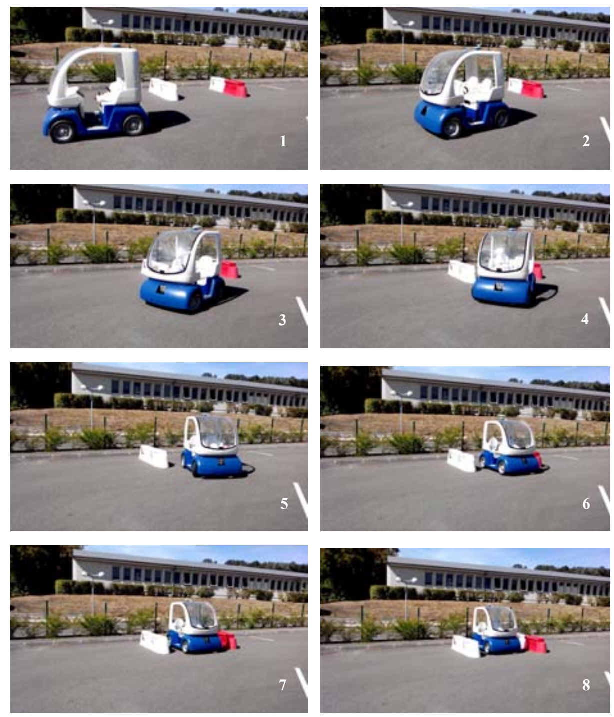

Figure 8. Automatic perpendicular parking of a CyCab vehicle

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